**Topics: Normal distribution, Functions of Random Variables**

1. The time required for servicing transmissions is normally distributed with *μ* = 45 minutes and *σ* = 8 minutes. The service manager plans to have work begin on the transmission of a customer’s car 10 minutes after the car is dropped off and the customer is told that the car will be ready within 1 hour from drop-off. What is the probability that the service manager cannot meet his commitment?
2. 0.3875
3. 0.2676
4. 0.5
5. 0.6987

**Ans –**

**Work will begin after 10 min so average time will be 55 min.**

**standardizing a normally distributed variable:**

**Z = (X - μ) / σ**

**Z = (60 - 55) / 8**

**Z = 5 / 8**

**Z = 0.625**

**1-pnorm(0.625)**

**= 0.030396**

**the probability that the service time exceeds 60 minutes (1 hour) is approximately 0.2659, or about 26.59 %. This is the probability that the service manager cannot meet his commitment to have the car ready within 1 hour from drop-off.**

1. The current age (in years) of 400 clerical employees at an insurance claims processing center is normally distributed with mean *μ* = 38 and Standard deviation *σ* =6. For each statement below, please specify True/False. If false, briefly explain why.
2. More employees at the processing center are older than 44 than between 38 and 44.

**Ans –**

**Z = (X - μ) / σ**

**Z = (44 – 38) / 6**

**Z = 6 / 6**

**Z = 1**

**norm.cdf(1) = 0.8413**

**People above 44 age = 100 – 84.13**

**​​​ = 15.87 %**

**​​​ = 63 out of 400**

**Z = (X - μ) / σ**

**Z = (38 – 38) / 6**

**Z = 0 / 6**

**Z = 0**

**norm.cdf(0) = 0.5**

**People between 38 and 44 age = 84.13 – 50**

**​​​​ = 34.13%**

**​​​​ = 137 out of 400**

**More employees at the processing center are older than 44 than between 34 and 44 is False.**

1. A training program for employees under the age of 30 at the center would be expected to attract about 36 employees.

**Ans -**

**Z = (X - μ) / σ**

**Z = (30 – 38) / 6**

**Z = -8 / 6**

**Z = -1.3333**

**norm.cdf(-1.33) = 0.09175913565028077**

**= 9.15%**

**= 36 out of 400**

**A training program for employees under the age of 30 at the center would be expected to attract about 36 employees is True.**

1. If *X1* ~ *N*(μ, σ2) and *X*2 ~ *N*(μ, σ2) are *iid* normal random variables, then what is the difference between 2 *X*1 and *X*1 + *X*2? Discuss both their distributions and parameters.

**Ans –**

**X1 ~ N(μ, σ^2), where X1 follows a normal distribution with mean μ and variance σ^2.**

**X2 ~ N(μ, σ^2), where X2 follows a normal distribution with the same mean μ and variance σ^2.**

**- X1 and X2 are independent and identically distributed (iid) random variables.**

**1. Difference between 2X1 and X1 + X2:**

**i) Distribution of 2X1:**

**If X1 follows a normal distribution N(μ, σ^2), then the random variable 2X1 will also follow a normal distribution. The mean of 2X1 will be 2μ (since you're multiplying by 2), and the variance will be 4σ^2 (since variance scales by the square of the constant).**

**So, 2X1 ~ N(2μ, 4σ^2).**

**ii) Distribution of X1 + X2:**

**Since X1 and X2 are independent and identically distributed, the sum of two independent normal random variables is itself a normal random variable. The mean of X1 + X2 will be 2μ (sum of individual means), and the variance of X1 + X2 will be 2σ^2 (sum of individual variances).**

**So, X1 + X2 ~ N(2μ, 2σ^2).**

**2. Comparison:**

**Both 2X1 and X1 + X2 are normal random variables, but they have different distributions and parameters:**

**- For 2X1:**

**- Distribution: N(2μ, 4σ^2)**

**- Mean: 2μ**

**- Variance: 4σ^2**

**- For X1 + X2:**

**- Distribution: N(2μ, 2σ^2)**

**- Mean: 2μ**

**- Variance: 2σ^2**

**Even though the mean of both 2X1 and X1 + X2 is the same (2μ), the variance and the distribution shapes differ between the two. The distribution of 2X1 has a higher variance than X1 + X2, meaning it is more spread out compared to X1 + X2. This difference in variance is due to the scaling effect of multiplication in 2X1, leading to a wider spread of values.**

1. Let X ~ N(100, 202). Find two values, *a* and *b*, symmetric about the mean, such that the probability of the random variable taking a value between them is 0.99.
2. 90.5, 105.9
3. 80.2, 119.8
4. 22, 78
5. 48.5, 151.5
6. 90.1, 109.9
7. Consider a company that has two different divisions. The annual profits from the two divisions are independent and have distributions Profit1 ~ N(5, 32) and Profit2 ~ N(7, 42) respectively. Both the profits are in $ Million. Answer the following questions about the total profit of the company in Rupees. Assume that $1 = Rs. 45
8. Specify a Rupee range (centered on the mean) such that it contains 95% probability for the annual profit of the company.

**Ans –**

**Company’s Profit ​= P ~ N (5 + 7, 32 + 42)**

**= N(12,52)**

**95% of the probability lies between 1.96 S.D. of the mean**

**= (12 – 1.96 \* 5, 12 + 1.96 x 5)**

**= (12 – 9.8, 12 + 9.8)**

**= (2.2, 21.8)**

**= (2.2 \* 45), (21.8 \* 45)**

**= 99 rs, 981 rs.**

1. Specify the 5th percentile of profit (in Rupees) for the company

**Ans –**

**= 0.05**

**P value = 0.05**

**From P value of Z score table**

**= -1.644**

**P = 12 – 8.22**

**P = 3.78**

**(3.78 \* 45 = 170.1)**

**5th percentile of profit is Rs 170.1 Million**

1. Which of the two divisions has a larger probability of making a loss in a given year?

**Ans –**

**The first division of company have larger probability of making loss in year.**